Government of **Western Australia**
School Curriculum and Standards Authority

WACE
Western Australian Certificate of Education

Western Australian Certificate of Education Examination, 2014

Question/Answer Booklet

MATHEMATICS: SPECIALIST 3C/3D

Section Two: Calculator-assumed

Place one of your candidate identification labels in this box.

Ensure the label is straight and within the lines of this box.

Student Number: In figures | | | | | | | | | |

In words _____

_____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for this section: one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Ref: 14-089
MAS3CD-S2

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	50	$33\frac{1}{3}$
Section Two: Calculator-assumed	12	12	100	100	$66\frac{2}{3}$
				Total	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2014*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

See next page

Section Two: Calculator-assumed **(100 Marks)**

This section has **12** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 100 minutes.

Question 9 **(7 marks)**

A line L passes through the point $(-4, 2, 2)$ in the direction $2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$.

(a) Determine the coordinates of the point where L intersects the plane Π with equation $2x + 3y - z = 6$. (4 marks)

(b) Determine the size of the angle between L and Π. (3 marks)

Question 10 **(5 marks)**

A survey of the passengers on a train on one day last month showed that:

- The sum of the adults, students, seniors and children was 274.
- The sum of the students and the seniors on the train was 2 more than the sum of the adults and children on the train.
- The number of students exceeded the number of adults by an amount equal to one-eighth of the number of children on the train.
- The sum of the adults and the seniors plus twice the number of students and children combined was 423.

Let x be the number of adults on the train and the corresponding numbers of students, seniors and children be y, z and w respectively.

(a) From the above information, construct a system of linear equations and write it as a matrix equation. (3 marks)

(b) How many people were in each group, adults, students, seniors and children, on the train? (2 marks)

See next page

Question 11 **(5 marks)**

An equilateral triangle has vertices A, B and C, where A is the point $\sqrt{3} - i$ in the Argand plane.

The circumcircle is drawn that passes through vertices A, B and C and has a centre inside the triangle, called the circumcentre.

The circumcentre of the triangle is located at the origin.

Find the complex numbers z_1 and z_2 corresponding to the vertices B and C, expressing your answer in exact Cartesian form.

See next page

Question 12 **(7 marks)**

A square matrix M of order 2 is such that $M^2 = 10M - 4I$, where I is the 2×2 identity matrix.

(a) Determine an expression for M^{-1} in the form $aI + bM$. (3 marks)

Now consider the matrix $P = \begin{bmatrix} p & 3p-1 \\ 1 & p \end{bmatrix}$.

(b) Determine the value(s) of p such that $\det(P) = \det(P^{-1})$. (4 marks)

See next page

Question 13 **(8 marks)**

The coordinates of the points A, B, C and D are (4, 1, –1), (3, 3, 5), (1, 0, p) and (1, 1, 2) respectively.

(a) Given that the vectors $\overrightarrow{BC}$ and $\overrightarrow{AC}$ are perpendicular, determine the value of p. (3 marks)

(b) Determine an equation for the line L, which passes through the point B and is parallel to the vector $\overrightarrow{AC}$. (2 marks)

(c) Determine a vector equation of the plane Π, which contains the line L and passes through the point D. (3 marks)

See next page

Question 14 **(9 marks)**

Two hundred rabbits in a region with an estimated population of 150 000 rabbits have a highly contagious disease.

The disease is known to spread at the weekly rate of 1% of the remaining healthy rabbits so that $\frac{dP}{dt} = 0.01(150\ 000 - P)$, where P is the number of rabbits that have already been infected.

(a) Determine the function $P(t)$ where t represents the time elapsed since the initial 200 cases were noted. (4 marks)

See next page

(b) How many rabbits become infected during the first week? (3 marks)

(c) After how many days does the total number of infected rabbits first exceed 2500? (2 marks)

See next page

Question 15 **(8 marks)**

A population of blue-tongued turtles is studied and the following data are collected on the first day of the experiment:

Age (months)	0–6	6–12	12–18	18–24
Initial population	4500	1800	900	130
Survival rate	0.5	0.8	0.4	0
Birth rate	0	1.9	1.5	0.7

(a) What is the total population six months after the start of the experiment? (3 marks)

(b) What is the total population 10 years after the start of the experiment? (2 marks)

See next page

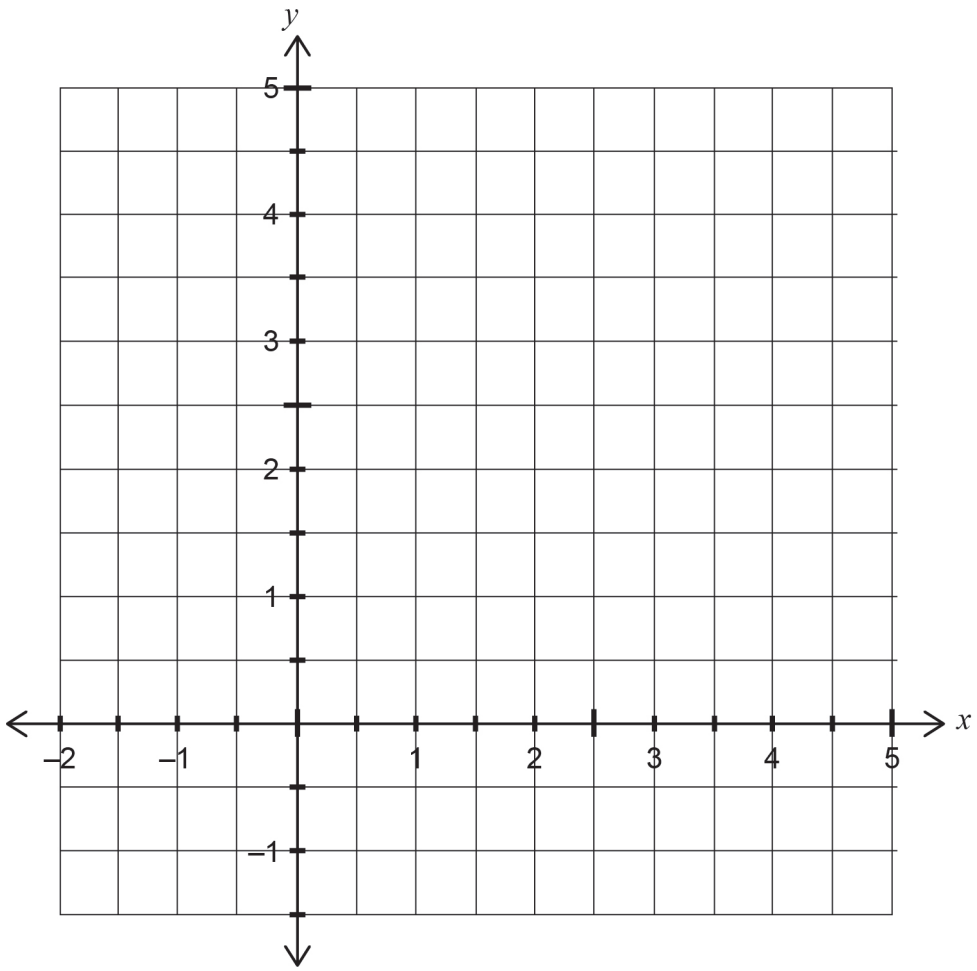
(c) In the long term, by what percentage does the population increase every six months?
(3 marks)

See next page

Question 16 **(13 marks)**

Let the function $f(x)$ be defined by $f(x) = 2e^{-x} - 1$.

(a) On the axes below sketch the graph of $f(x)$. (2 marks)



(b) Determine the defining rule for the inverse function $f^{-1}(x)$. (2 marks)

See next page

(c) Sketch the graph of $f^{-1}(x)$ on the axes on the previous page. Label your sketch clearly, indicating the coordinates of the point(s) of intersection of f and f^{-1}. (5 marks)

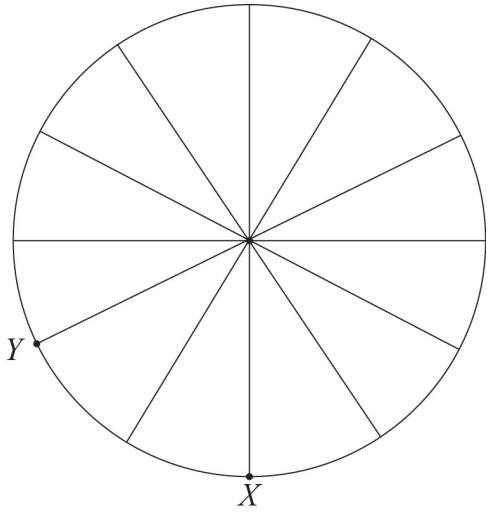
(d) Determine the area of the region bounded by the graphs of f and f^{-1}. (4 marks)

See next page

Question 17 **(9 marks)**

The world's first Ferris wheel (a rotating, upright wheel) was built in 1893.

A model of the wheel is to be built. Two of the passenger cars on the rim of the model are to be painted gold. These cars are labelled X and Y on the diagram below.



This model wheel has a diameter of 5 m and rotates clockwise at the rate of one revolution every 8 seconds. The twelve spokes are evenly spaced and the wheel begins to turn when X is at ground level at the bottom of the wheel. After t seconds the point X is a height $h(t)$ metres above the ground.

(a) Write an expression for $h(t)$ in the form $h(t) = a + b\cos(nt)$ and hence evaluate a, b and n using the information above. (4 marks)

See next page

(b) Express the acceleration $\frac{d^2h}{dt^2}$ in terms of h. (2 marks)

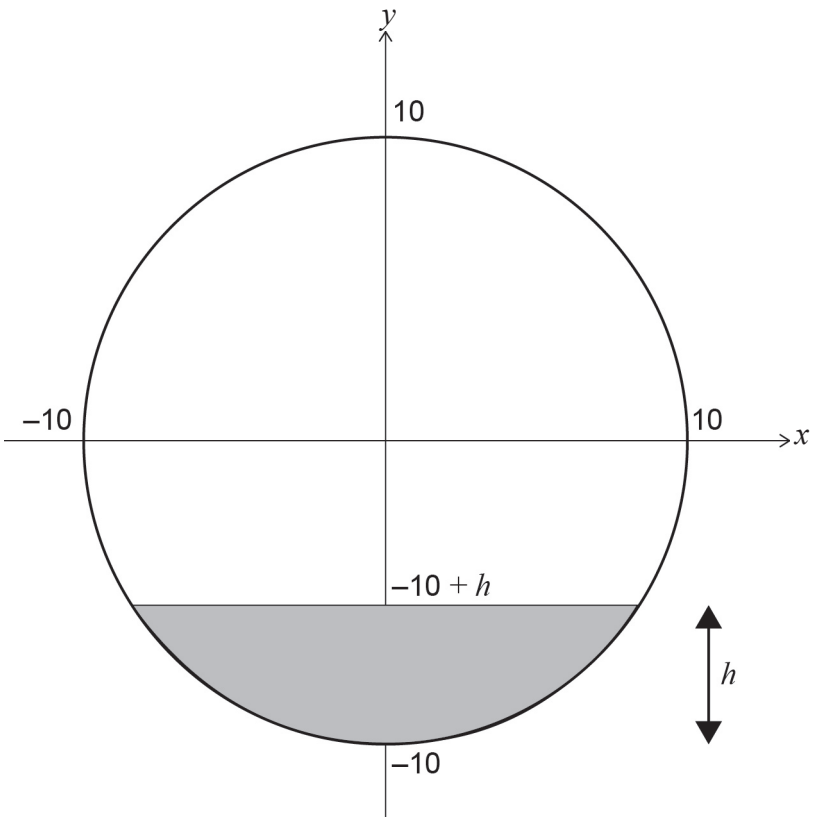
(c) Describe the position of the car X when its vertical acceleration is largest. (1 mark)

(d) At what time are both X and Y first at the same height? What is their height at that time? (2 marks)

See next page

Question 18 **(8 marks)**

A container is made by rotating about the y-axis that part of the circle $x^2 + y^2 = 100$ for which $-10 \leq y \leq -10 + h$.



(a) The volume of the solid of revolution about the y-axis is given by $V = \int \pi x^2 dy$. Show that the volume of the container is $V = \frac{\pi h^2}{3}(30 - h)$. (4 marks)

A hemispherical bowl of radius 10 cm forms part of a display in the window of a chocolate shop.

The axis of the bowl is vertical and the bowl is filled from empty with warm chocolate at a constant rate such that the chocolate begins to overflow after 20 minutes.

(b) Calculate the rate of change in the depth of chocolate in the container when the depth in the middle of the bowl is 5 cm. (4 marks)

See next page

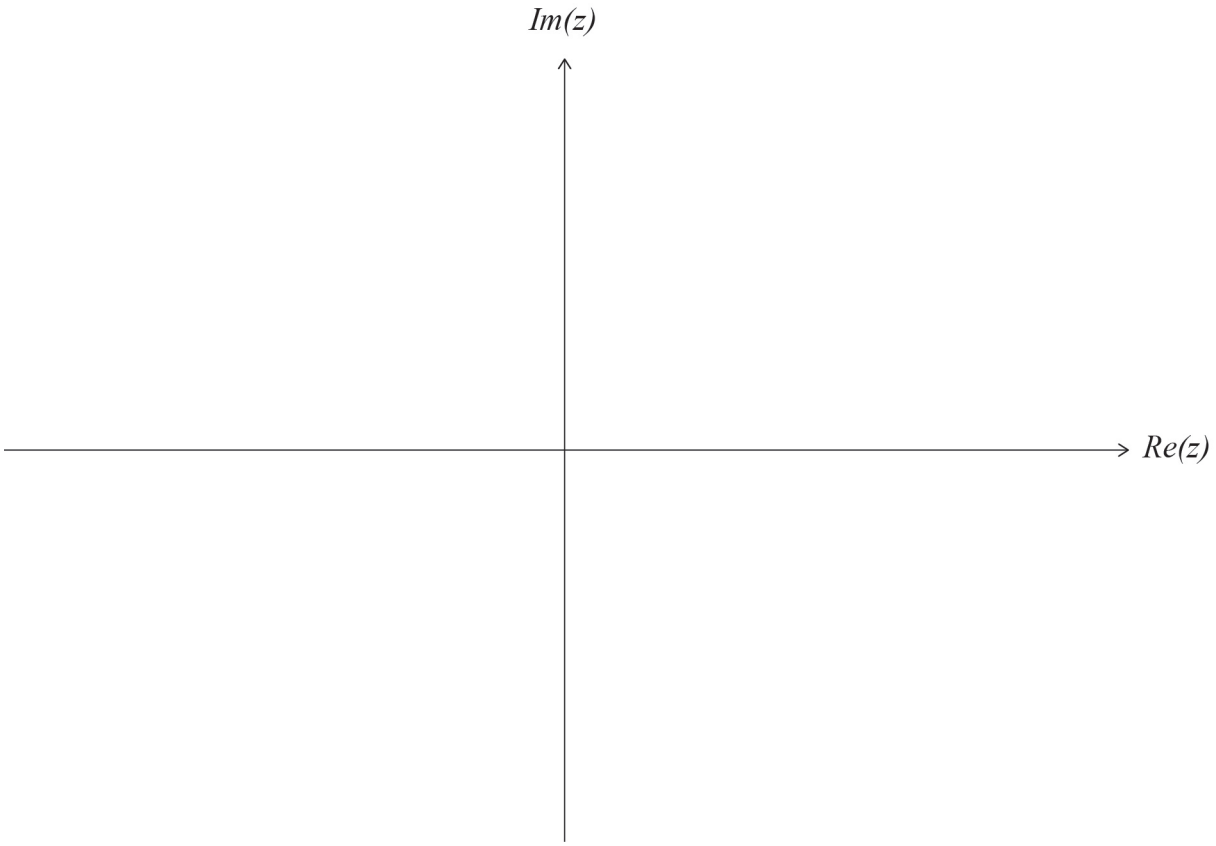
Question 19 **(11 marks)**

(a) Consider the following two sets of complex numbers z:

$S: \ |z - 1 - 3i| = 3$

$T: \ \frac{\pi}{2} \leq \arg(z - 1 - 3i) \leq \pi.$

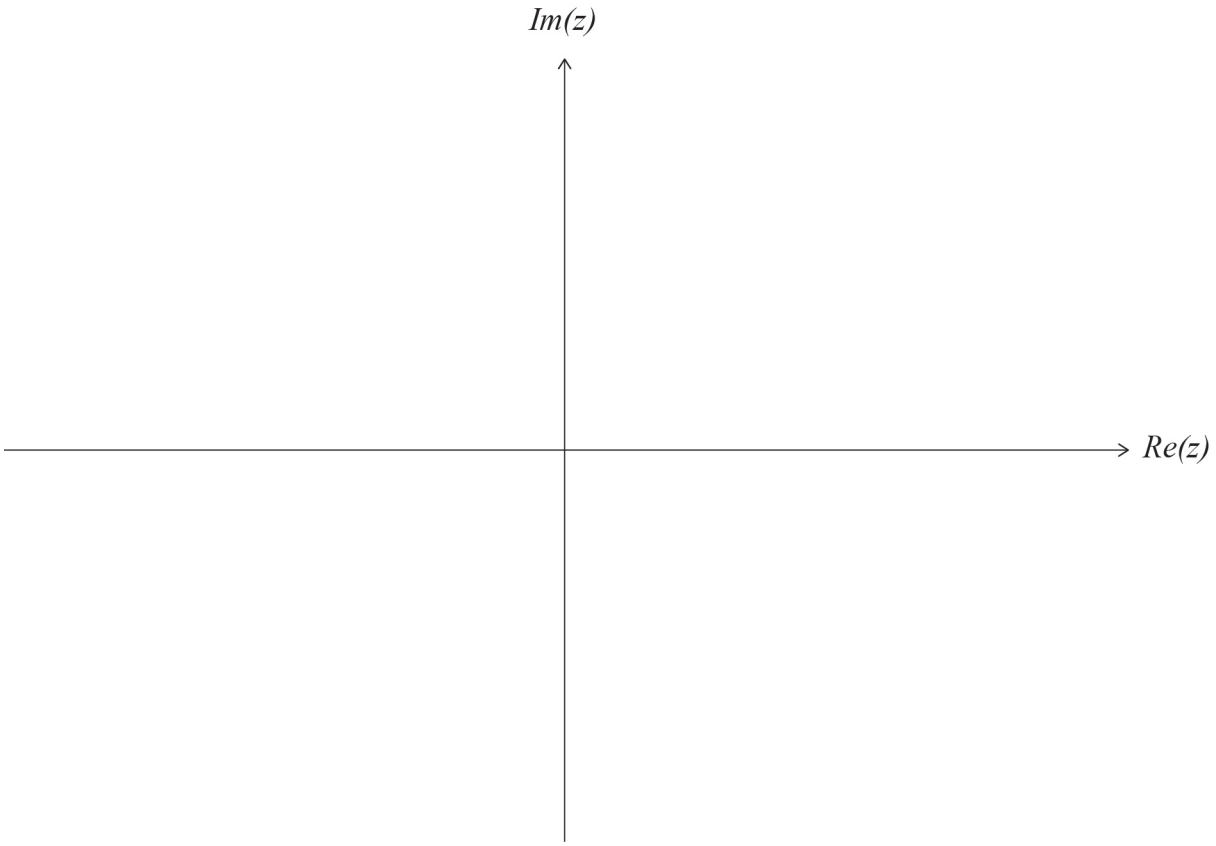
(i) Sketch the above sets of complex numbers on the Argand plane below. (4 marks)



(ii) Describe, in Cartesian terms, the intersection of these two sets.

(2 marks)

(b) Sketch $\left\{ z : z = 3e^{-i\pi t} \text{ and } 0 < t \leq \frac{3}{2} \right\}$ on the Argand plane below. (5 marks)



See next page

Question 20 **(10 marks)**

The Fibonacci sequence $F_0, F_1, F_2, \ldots$ is defined by $F_0 = 0, F_1 = 1$ and, for $n \geq 0$,

$$F_{n+2} = F_n + F_{n+1}.$$

Define the sequence $G_k = F_{k+2} F_k - (F_{k+1})^2$ for $k \geq 0$.

(a) Evaluate G_0, G_1 and G_2. (2 marks)

(b) (i) Prove that $G_{k+1} + G_k = 0$. (3 marks)

(ii) Make a conjecture as to the value of G_k. Use induction to prove your conjecture. (3 marks)

See next page

(c) By taking the tangent of both sides, or otherwise, prove that

$$\tan^{-1}\left(\frac{1}{F_{2n}}\right) - \tan^{-1}\left(\frac{1}{F_{2n+2}}\right) = \tan^{-1}\left(\frac{1}{F_{2n+1}}\right).$$

(2 marks)

End of questions

Additional working space

Question number: _____

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Question number: _____

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